Testing a New Lattice Heavy Quark Action

Jon A. Bailey[1], Carleton DeTar[2], Yong-Chull Jang[1],
Andreas Kronfeld[3], Weonjong Lee[1], M. B. Oktay[2]

Lattice Gauge Theory Research Center,
Seoul National University
(Fermilab Lattice[1], MILC[2], and SWME[3] Collaborations)

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Flavor Physics and Lattice Calcuation with Heavy Quarks

- In flavor physics, we are interested in the CKM matrix element V_{cb}.
- Combining HFAG average of experimental results and lattice form factor $\mathcal{F}$ calculation of the semi-leptonic decays, we can extract exclusive V_{cb}.

$$\bar{B} \to D^* l \nu_l \,, \; \bar{B} \to D l \nu_l$$

- Because the dominant error for the form factor $\mathcal{F}$ calculation is heavy quark discretization error, we need an highly improved lattice action or finer lattice ensemble.
- OK action was designed as an improved action.

Discretization Effects on the Lattice Action

$$S_{\text{Wilson}} = \sum_x \bar{\psi}(x)\left[\gamma_4 D_4 + m_0 + \vec{\gamma}\cdot\vec{D}\right]\psi(x) - \frac{1}{2} r_s a \sum_x \bar{\psi}(x)\left[\triangle_4 + \triangle^{(3)}\right]\psi(x)$$

- What is the size of discretization error?
- How can we reduce the discretization error even with a finite lattice spacing $a \neq 0$?

Symanzik Effective Action

$$\mathcal{L}_{\text{LGT}} \doteq \mathcal{L}_{\text{sym}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_I$$

$$\mathcal{L}_{\text{QCD}} = -\bar{q}\left[\gamma_4 D_4 + m + \gamma \cdot \boldsymbol{D}\right] q$$

$$\mathcal{L}_I = \sum_i a^{\dim L_i - 4} K_i(ma, g^2; c_j; \mu a)\mathcal{L}_i \sim \sum_i \mathcal{O}(a\Lambda)^{\dim L_i - 4}$$

- $\mathcal{L}_i$ can be treated as a perturbation, if $a\Lambda < 1$.
- $K_i(ma, g^2; c_j; \mu a) = g^{2k} \sum_m g^{2m} K_i^{[m]}(ma; c_j; \mu a)$ and $K_i^{[m]}$ can be further expanded by ma if we consider a light quark $ma \ll 1$.
- The lattice artifact can be controlled by requiring the condition $K_i^{[m]} = 0$ up to a given order of g^2 and a: $\mathcal{O}(a^n)$ improvement.
- What modifications are needed if we want to treat a heavy quark $ma \geq 1$ discretization effects systematically?

HQET Inspired Symanzik-like Improvement

$$\mathcal{L}_{\text{LGT}} \doteq \bar{\mathcal{L}}_{\text{QCD}} + \bar{\mathcal{L}}_I$$

$$\bar{\mathcal{L}}_{\text{QCD}} = -\bar{Q}\left[\gamma_4 D_4 + m_1 + \sqrt{m_1/m_2}\gamma\cdot\boldsymbol{D}\right]Q$$

$$\bar{\mathcal{L}}_I = \sum_i a^{\dim\bar{L}_i - 4}\bar{K}_i(m_2 a, g^2; c_j; \mu a)\bar{\mathcal{L}}_i \sim \sum_i \mathcal{O}(a\boldsymbol{p})^{\dim\bar{L}_i - 4}$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

- $\bar{\mathcal{L}}_i$ does not contain time derivative.
- All dependences on the (heavy) quark mass are isolated in the short-distance coefficients $\bar{K}_i(m_2 a, g^2; c_j; \mu a)$.
- $\bar{\mathcal{L}}_i$ can also be treated as a perturbation, if $a\boldsymbol{p} < 1$.
- $\bar{K}_i = 0\,;\ a\boldsymbol{p} < 1\,,\ m_2 a \geq 1$ yields non-relativistic interpretation of $\bar{\mathcal{L}}_{\text{QCD}} \doteq \mathcal{L}_{\text{HQET}}$ with a mistuned $m_1 \neq m_Q$, $(m_2 = m_Q)$.

Fermilab Action

- Fermilab action is the Sheikholeslami-Wohlert "clover" action with a non-relativistic interpretation (Fermilab formulation).

$$S_{\text{Fermilab}}(m_Q = m_2(m_0)) = S_0(\zeta = 1) + S_B + S_E$$

$$\lambda \sim a\Lambda,\ \Lambda/m_Q$$

$$S_0 = m_0 \sum_x \bar{\psi}(x)\psi(x) + \sum_x \bar{\psi}(x)\gamma_4 D_4 \psi(x) \qquad : \mathcal{O}(1)$$

$$+ \zeta \sum_x \bar{\psi}(x)\vec{\gamma}\cdot\vec{D}\psi(x) - \tfrac{1}{2}a \sum_x \bar{\psi}(x)\triangle_4\psi(x) - \tfrac{1}{2}r_s\zeta a \sum_x \bar{\psi}(x)\triangle^{(3)}\psi(x)$$

$$S_B = -\tfrac{1}{2}c_B\zeta a \sum_x \bar{\psi}(x) i\vec{\Sigma}\cdot\vec{B}\psi(x) \qquad : \mathcal{O}(\lambda)$$

$$S_E = -\tfrac{1}{2}c_E\zeta a \sum_x \bar{\psi}(x)\vec{\alpha}\cdot\vec{E}\psi(x) \qquad : \mathcal{O}(\lambda^2)$$

[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action

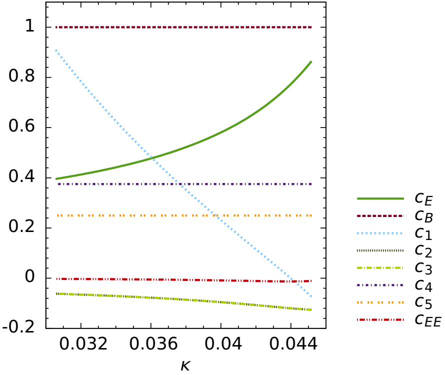
- OK action includes dim-6 and -7 operators necessary for tree-level matching to QCD through order $\mathcal{O}(\lambda^3)$: $S_{\mathrm{OK}} = S_{\mathrm{Fermilab}} + S_{\mathrm{new}}$

$$
\begin{aligned}
S_{\mathrm{new}} = {} & c_1 a^2 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \triangle_i \psi(x) \\
& + c_2 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi(x) \\
& + c_3 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi(x) \\
& + c_{EE} a^2 \sum_x \bar{\psi}(x) \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi(x) \\
& + c_4 a^3 \sum_x \bar{\psi}(x) \sum_i {\triangle_i}^2 \psi(x) \\
& + c_5 a^3 \sum_x \bar{\psi}(x) \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi(x) \qquad : \mathcal{O}(\lambda^3)
\end{aligned}
$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

OK Action

Tree-level Matching



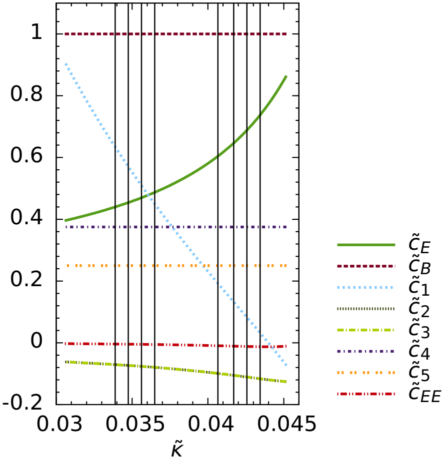
$$\frac{1}{2\kappa_t} = m_0 a + 1 + 3 r_s \zeta + 18 c_4$$

$$r_s = \zeta = 1$$

- All coefficients are bounded over a mass range.
- $c_B = c_E = 1$ for the Fermilab action.

OK Action

Tadpole Improvement



- $U_\mu(x) \to U_\mu(x)/u_0$

$$\frac{1}{2\kappa_t} = u_0(\tilde{m}_0 a + 1 + 3r_s\zeta + 18\tilde{c}_4) = \frac{u_0}{2\tilde{\kappa}_t}$$

$$c_E = \tilde{c}_E u_0^{-3}$$

$$c_{EE} = \tilde{c}_{EE} u_0^{-4}$$

$$c_3 = \tilde{c}_3 u_0^{-4}$$

Meson Correlator

$$C(t, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^\dagger(t, \boldsymbol{x}) \mathcal{O}(0, \boldsymbol{0}) \rangle$$

- On the lattice, we calculate the 2-point correlator.
- 11 meson momenta $|\boldsymbol{p}a| (= 0, 0.099, \cdots, 1.26)$ for dispersion fit
- MILC asqtad $N_f = 2 + 1$ ensemble

a(fm)	$N_L^3 \times N_T$	β	am_l'	am_s'	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	500	6

Interpolating Operator

- Valence heavy quark $\psi(x)$: (tadpole improved) OK action with

$$\kappa_{\text{OK}} = 0.039, 0.040, 0.041, 0.042, 0.0468, 0.048, 0.049, 0.050$$

and Fermilab action with

$$\kappa_{\text{FNAL}} = 0.083, 0.091, 0.121, 0.127$$

- Valence light quark $\chi(x)$: asqtad staggered action $(am_q = am_s')$
- Heavy-light meson interpolating operator

$$\mathcal{O}_{\text{t}}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta\text{t}}(x)\chi(x)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(pseudoscalar)} \\ \gamma_\mu & \text{(vector)} \end{cases}, \quad \Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Quarkonium interpolating operator

$$\mathcal{O}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

Correlator Fit: extract the energy E

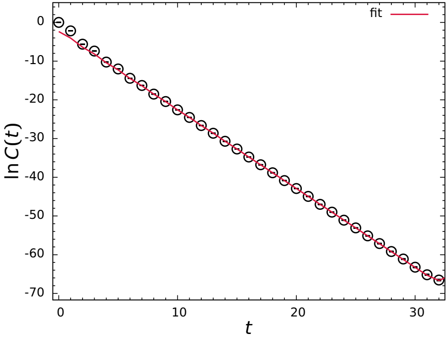
OK action, $\overline{Q}q$, PS, $\kappa = 0.041$, $p = 0$

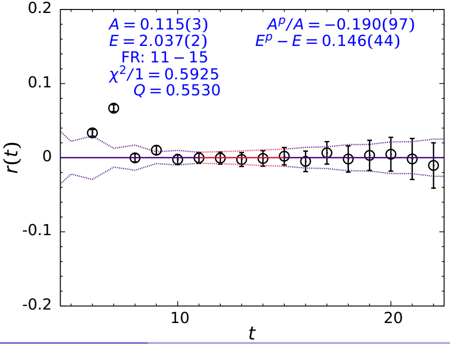
- fit function (T: time extent of lattice)

$$f(t) = A\left\{e^{-Et} + e^{-E(T-t)}\right\} + (-1)^t A^p \left\{e^{-E^p t} + e^{-E^p(T-t)}\right\}$$

- fit residual

$$r(t) = \frac{C(t) - f(t)}{|C(t)|}, \text{where } C(t) \text{ is data.}$$





Correlator Fit: extract the energy E

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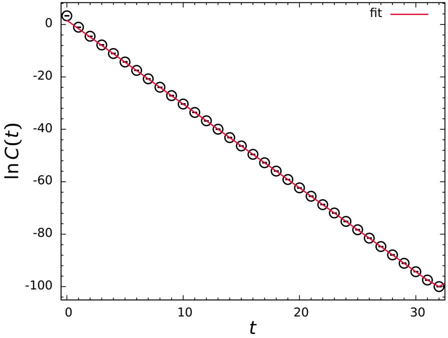
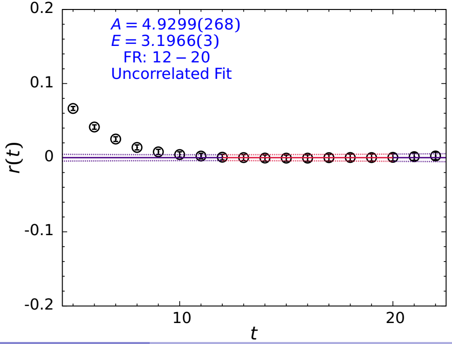


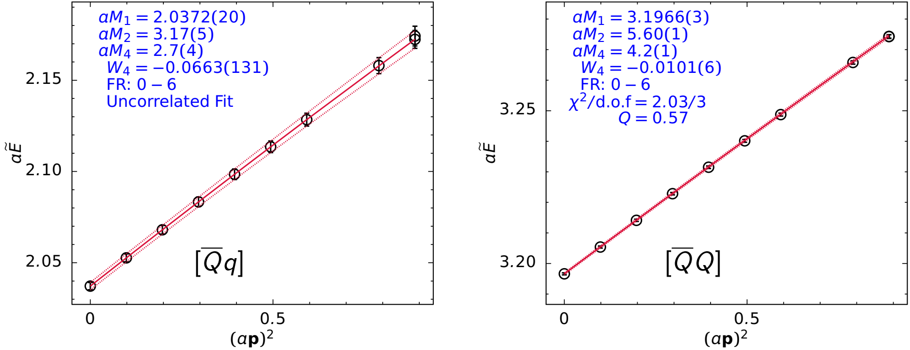


Dispersion Fit: extract the masses M_1 and M_2

OK action, PS, $\kappa = 0.041$

$$\text{fit: } E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4 \;\Rightarrow\; \text{plot: } \widetilde{E} = E + \frac{a^3 W_4}{6}\sum_i p_i^4$$

- Two points with momentum $\boldsymbol{n} = (2,2,1)\,,(3,0,0)$ are distinguishable to the rotation symmetry breaking W_4 term.



Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

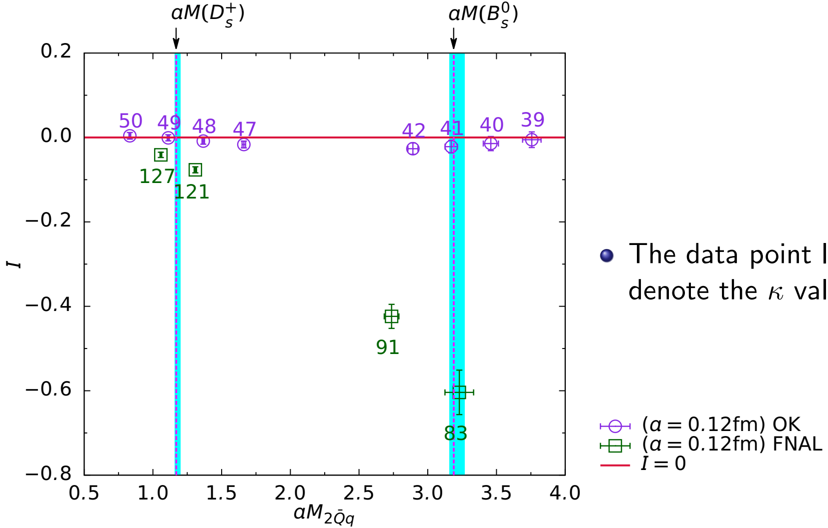
$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- By design, the inconsistency parameter I can examine the action improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms. I isolate the δB of $\mathcal{O}(\boldsymbol{p}^2)$ effect.
- In the continuum limit, $B_1 = B_2$ and I vanishes.
- By including up to $\mathcal{O}(\boldsymbol{p}^4)$, the OK action is closer to the renormalized trajectory S_{RT} than the Fermilab action.
- We expect I is close to 0.

Improvement Test: Inconsistency Parameter

- Near B_s^0 mass, the coarse ($a = 0.12$fm) ensemble data shows significant improvement compared to the Fermilab action.



- The data point labels denote the κ values.

Improvement Test: Hyperfine Splitting Δ

- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

$$\Delta_1 = M_1^* - M_1\,,\ \Delta_2 = M_2^* - M_2$$

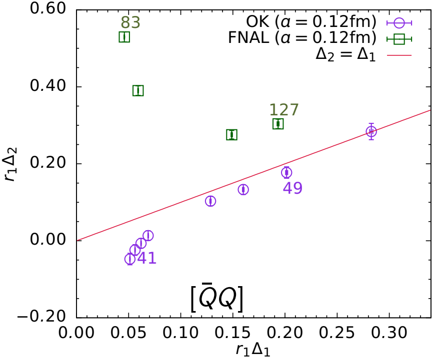
$$M^{(*)}_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B^{(*)}_{1\overline{Q}q}$$
$$M^{(*)}_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B^{(*)}_{2\overline{Q}q}$$
$$\delta B^{(*)} = B_2^{(*)} - B_1^{(*)}$$

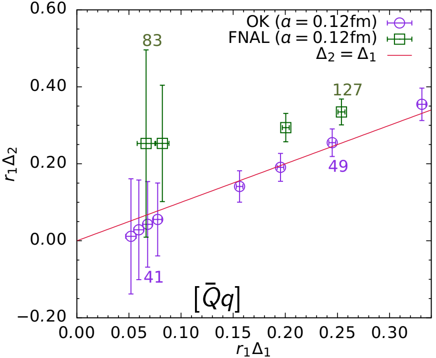
$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

- In the continuum limit $\Delta_2 = \Delta_1$.

Improvement Test: Hyperfine Splitting Δ

- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- The heavy-light results also shows an improvement near charm region; for bottom region, it is not clear to say an improvement. However, we expect that OK action gives an improvement in the bottom region ($r_1\Delta_1$).





Conclusions

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\boldsymbol{p}^4)$ terms.
- Smaller error on the kinetic mass M_2 implies that we can tune κ more precisely.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- Hyperfine splitting for the heavy-light system shows a clear improvement near D_s.

Applications and Outlook

- We plan to calculate form factor of the decay $\bar{B} \to D^* l\nu$ by using OK action. Then, we can determine exclusive V_{cb} with higher precision.
- Lattice current improvement to the same order of the OK action is under way.
- We plan on the 1-loop improvement for the coefficients c_B and c_E in the OK action.
- We plan on the development of highly optimized CG inverter using QUDA (GPU computing).

Thank you for your attention.

Improvement Test: Inconsistency Parameter

- Considering non-relativistic limit of quark and anti-quark system, for S-wave case ($\mu_2^{-1} = m_{2\overline{Q}}^{-1} + m_{2q}^{-1}$),

$$\begin{aligned}\delta B_{\overline{Q}q} &= \frac{5}{3}\frac{\langle \boldsymbol{p}^2\rangle}{2\mu_2}\left[\mu_2\left(\frac{m_{2\overline{Q}}^2}{m_{4\overline{Q}}^3} + \frac{m_{2q}^2}{m_{4q}^3}\right) - 1\right] \quad (m_4 : c_1, c_3) \\ &+ \frac{4}{3}a^3\frac{\langle \boldsymbol{p}^2\rangle}{2\mu_2}\mu_2\left(w_{4\overline{Q}}m_{2\overline{Q}}^2 + w_{4q}m_{2q}^2\right) \quad (w_4 : c_2, c_4) \\ &+ \mathcal{O}(p^4)\end{aligned}$$

[A. S. Kronfeld, NPB **53**, 401 (1997) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

- Leading contribution of $\mathcal{O}(\boldsymbol{p}^2)$ in δB vanishes when $w_4 = 0$, $m_2 = m_4$, not only for S-wave states but also for higher harmonics.